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Assignment no. 7
Bernoull's applications II

A. Cavitation & Syphon:

1. $Q = ?$, $P_1 = 275$ kpa (gage) , $P_{atm.} = 101$ kpa (abs.)
, $P_v = 5.2$ kpa (abs.)

Sol.:

* Before cavitation begin:

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gH}$$

$$A_1 = \frac{\pi}{4}(0.031)^2 = 7.55 * 10^{-4} \, m^2$$

$$, A_2 = \pi/4 \, (0.019)^2 = 2.835 * 10^{-4} \, m^2$$

$$, H = \frac{P_1 - P_2}{\gamma} + Z_1 - Z_2 \quad , \ldots \text{ datum at centerline} \rightarrow \therefore Z_1 = Z_2 = 0$$

$$\therefore H = \frac{P_1 - P_2}{\gamma} = \frac{735 - 550}{9.81} = 18.86 \, m$$

$$\therefore Q = \frac{(7.55*10^{-4})(2.835*10^{-4})}{\sqrt{(7.55*10^{-4})^2 - (2.835*10^{-4})^2}} \sqrt{2g(18.86)}$$

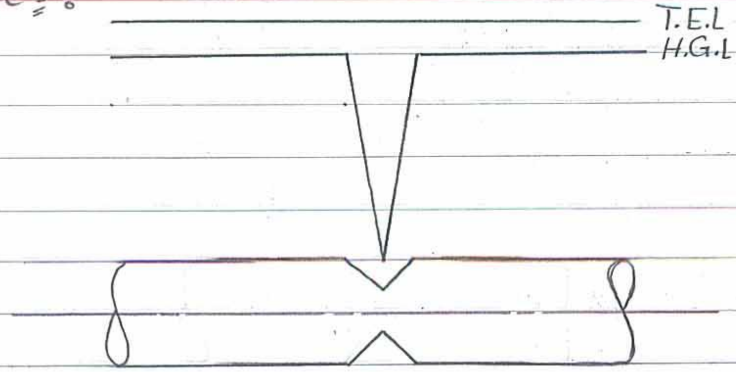
$$= 5.884*10^{-3} \, m^3/s$$

* After cavitation begin:

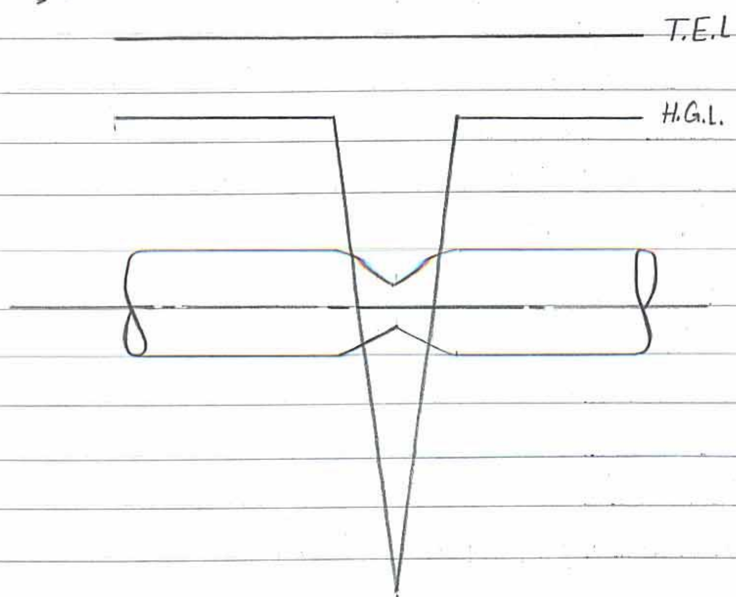
$$P_2 (\text{gage}) = P_v (\text{abs.}) - P_{atm.}$$
$$= 5.2 - 101 = -95.8 \text{ kpa}$$

$$\therefore Q = \frac{(7.55*10^{-4})(2.835*10^{-4})}{\sqrt{(7.55*10^{-4})^2 - (2.835*10^{-4})^2}} \sqrt{2g\left(\underbrace{\frac{735 - (-95.8)}{9.81}}_{\rightarrow H}\right)}$$

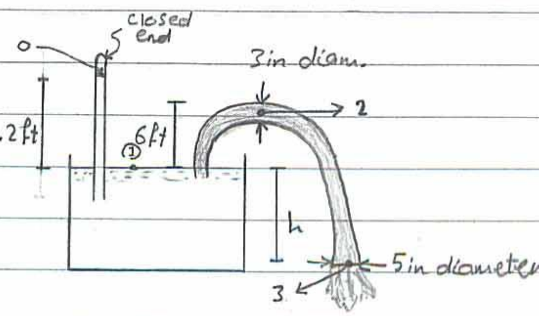
Case 1:



Case 2:



2. Sol.:



$\frac{P_1}{\gamma} = 30.2 + P_0/\gamma$, $\because P_1 = 0$ (gage)

$\therefore P_0/\gamma = -30.2$ ft (gage)

$\because P_0 = P_v$

$\therefore P_v/\gamma = -30.2$ ft

$\therefore \frac{P_2}{\gamma} = -30.2$ ft

Applying B.E. at points ①, ②, datum at ①:

$$0+0+0 = -30.2 + \frac{v_2^2}{2g} + 6$$

$\rightarrow v_2 = 39.47$ ft/s

$\therefore Q = \frac{\pi}{4}\left(\frac{3}{12}\right)^2(39.47) = 1.94$ ft^3/sec

$\rightarrow v_3 = \frac{Q}{A_3} = \frac{1.94}{\frac{\pi}{4}\left(\frac{5}{12}\right)^2} = 14.23$ m/s

Applying B.E. at point ① & ③, datum at ③:

$$0+0+h = 0 + \frac{14.23^2}{2g} + 0$$

$\therefore$ **h = 3.146 ft**

B. Pumps, turbines & Free Jets:

3. Sol.:

Applying B.E. at points ① & ②:

$$0+0+56+\overset{H_P}{24} = 0+\frac{U_2^2}{2g}+59+8\frac{U_{15}^2}{2g}+15\frac{U_{10}^2}{2g}$$

$\because Q = AU$

$$\therefore \quad 21 = \frac{Q^2}{2gA_2^2}+8\frac{Q^2}{2gA_{15}^2}+15\frac{Q^2}{2gA_{10}^2}$$

$$A_2 = \frac{\pi}{4}(0.075)^2 = 4.42*10^{-3}\ m^2$$

$$, A_{15} = \frac{\pi}{4}(0.15)^2 = 0.0176\ m^2$$

$$, A_{10} = \frac{\pi}{4}(0.1)^2 = 7.854*10^{-3}\ m^2$$

$$\therefore 21 = \frac{Q^2}{2g(4.42*10^{-3})^2}+8*\frac{Q^2}{2g(0.0176)^2}+15*\frac{Q^2}{2g(7.854*10^{-3})^2}$$

$$\rightarrow 21 = 16319.197\ Q^2 \rightarrow \boxed{Q = 0.0358\ m^3/s}$$

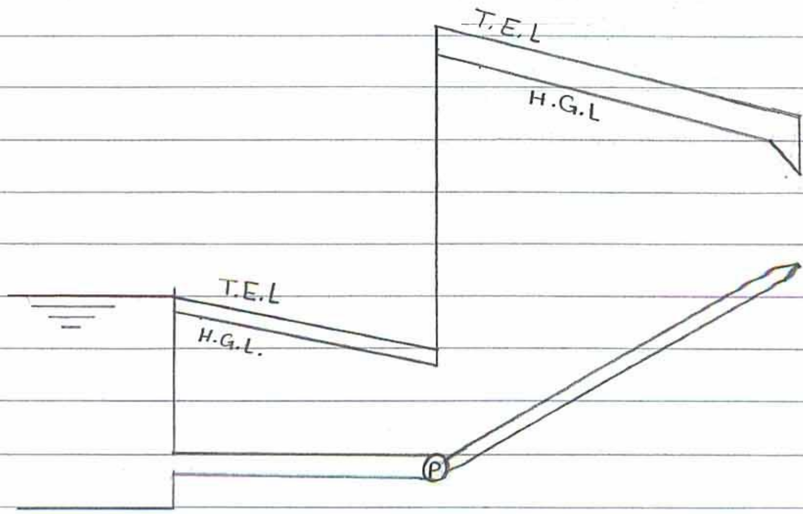
Applying B.E. at points ① & Ⓟ:

$$0+0+56 = \frac{P_s}{\gamma}+\frac{U_{15}^2}{2g}+50+8*\frac{U_{15}^2}{2g}$$

$$, U_{15} = \frac{Q}{A_{15}} = \frac{0.0358}{\frac{\pi}{4}(0.15)^2} = 2.026\ m/s$$

$$\therefore 56 = \frac{P_s}{\gamma}+\frac{(2.026)^2}{2g}+50+8*\frac{(2.026)^2}{2g}$$

$$\therefore \boxed{\frac{P_s}{\gamma} = 3.98\ m}$$



4. Sol. :

In the Shown exist two piezometers expressed the static head:

$$\frac{P_1}{\gamma} + z_1 = 100 \quad , \quad \frac{P_2}{\gamma} + z_2 = 200$$

Applying B.E. at Points ① & ② :

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + H_p = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$100 + \frac{v_1^2}{2g} + H_p = 200 + \frac{v_2^2}{2g} \longrightarrow ①$$

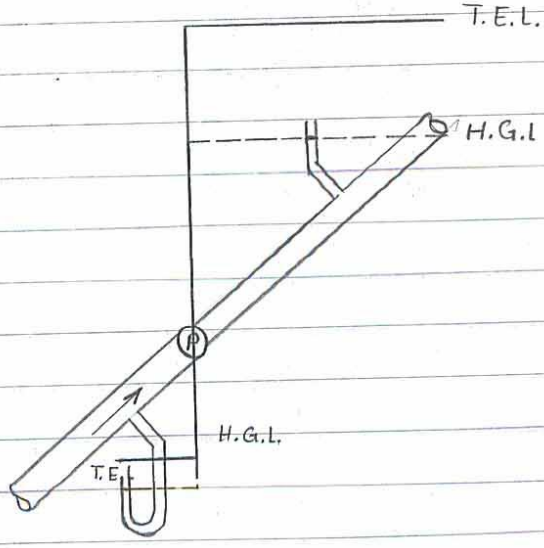
$$\because Q = A_1 v_1 \longrightarrow \therefore 1 = \frac{\pi}{4}\left(\frac{6}{12}\right)^2 v_1 \longrightarrow v_1 = 5.093 \text{ ft/s}$$

$$, \ 1 = \frac{\pi}{4}\left(\frac{4}{12}\right)^2 v_2 \longrightarrow v_2 = 11.46 \text{ ft/s}$$

$$\therefore 100 + \frac{5.093^2}{2g} + H_p = 200 + \frac{11.46^2}{2g}$$

$$\therefore H_p = 101.636 \text{ ft}$$

$$HP = \frac{\gamma Q H_p}{550} = \frac{62.4(1)(101.636)}{550} = 11.53 \ HP$$



5. Sol.:

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6. Sol.:

$\because v_y^2 = v_{y_0}^2 - 2g\,y_{max.}$

$\therefore 0 = v_{y_0}^2 - 2g(6) \longrightarrow \therefore v_{y_0} = 10.85 \; m/s$

$\because v_2 \sin(45^\circ) = v_{y_0}$

$\therefore v_2 = 15.344 \; m/s$

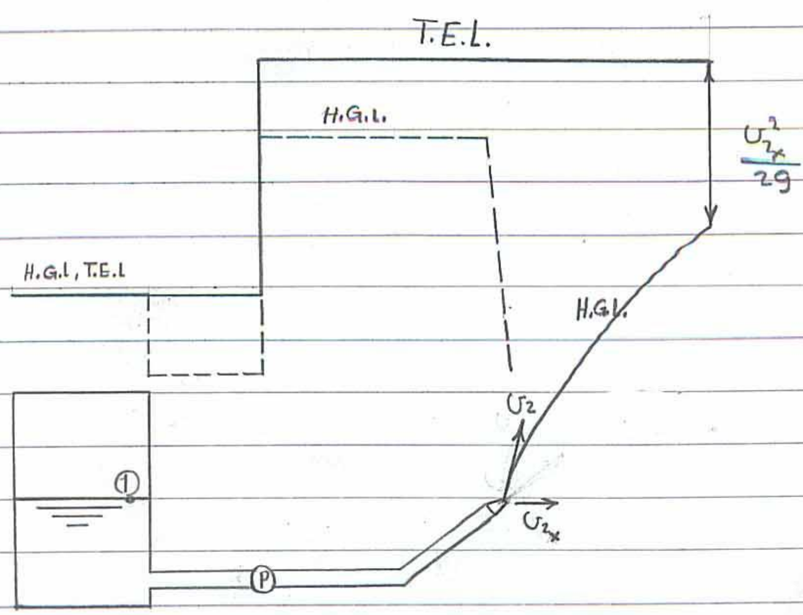
applying B.E. at points ① & ② , datum at ①:

$$\frac{35}{9.81} + 0 + 0 + H_p = 0 + \frac{(15.344)^2}{2g} + 0$$

$\therefore H_p = 8.432 \; m$

$$HP = \frac{\gamma Q H_p \eta}{K} = \frac{9810(15.344 \times \frac{\pi}{4} 0.075^2)(8.432)(0.7)}{735}$$

$= 5.3$ horse power



7. sol.:

$\because U_x = 15\, m/s$

$\therefore U_{x_2} = U_x = 15\, m/s$

$, \because U_2 \cos(30^\circ) = U_{x_2}$

$\therefore U_2 = 17.32\, m/s$

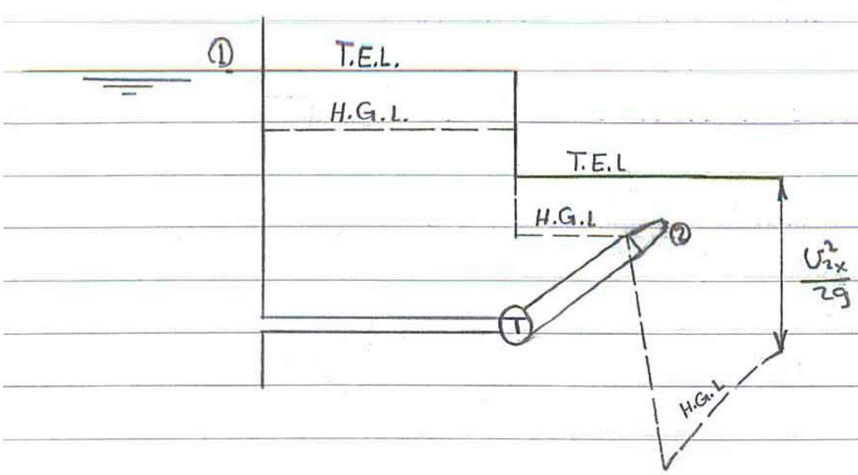
Applying B.E. bet. boints ① , ② :

$$0 + 0 + 75 - H_T = 0 + \frac{17.32^2}{2g} + 30$$

$\therefore H_T = 29.71\, m$, $Q = 17.32 * \frac{\pi}{4}(0.05)^2 = 0.034\, m^3/s$

$\therefore P_T = \gamma Q H_T = 9810 * (0.034)(29.71)$

$= 9909.473\ J/s$



C. Flow over notches:

8. $a = 25\ cm^2$, $C_{d_t} = 0.64$, $h_t = 3m$, $b_n = 0.5m$, $H_n = 0.6m$

$C_{d_n} = ?$

Sol.:

$$U_1 = \sqrt{2gh_t}$$
$$= \sqrt{2g(3)} = 7.672\ m/s$$

$\therefore Q = C_{d_t} a U_1 = (25*10^{-4})(7.672)*0.64 = 0.0123\ m^3/s$

$\because Q = \frac{2}{3} C_{d_n} b \sqrt{2g}\, H_n^{3/2}$

$\therefore 0.0123 = \frac{2}{3} C_{d_n} (0.5) \sqrt{2g}\, (0.6)^{3/2}$

$\longrightarrow C_{d_n} = 0.0179$

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9. Sol.:

$$Q = \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\, H^{5/2}$$

$$= \frac{8}{15} (0.6) \tan(30^\circ) \sqrt{2g}\, (0.4)^{5/2} = 0.083\ m^3/s$$

, $dQ = \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g} * \frac{5}{2} H^{3/2}\, dh$

$$= \frac{8}{15} (0.6) \tan(30) \sqrt{2g} * \frac{5}{2} (0.4)^{3/2} (0.004) = 2.07*10^{-3}$$

$\therefore \% Error = \frac{dQ}{Q} * 100 = \frac{2.07*10^{-3}}{0.083} * 100 = 2.5\%$

10. Sol.:

$$Q_{tot.} = Q_{rec.} + Q_V$$

$$= \frac{2}{3} C_d b \sqrt{2g}\, H^{3/2} + \frac{1}{2} * \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\, H^{5/2}$$

$C_d = 1$

$$\therefore Q_{tot.} = \frac{2}{3} \left(\frac{2}{10}\right) \sqrt{2g}\, (0.15)^{3/2} + \frac{1}{2} * \frac{8}{15} \tan(30) \sqrt{2g}\, (0.15)^{5/2}$$

$$= 0.0402\ m^3/s$$